A Simple Bargaining Model on Friendly and Hostile Takeovers

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Abstract

We propose a bargaining model that characterizes conditions for an either friendly, hostile or unsuccessful takeover. These conditions are related to private benefits of control and selling incentives for the target management, the takeover premium, the toehold size, and the level of breakup fees. Conditions for an efficient takeover are also established. An alternative modeling of hostility and the adoption of a negotiation - rather than an auction setup as most of prior literature - generate new insights on the evolution of corporate governance in the US and deliver testable implications for its market for corporate control consistent with available evidence.

Key words: takeover, negotiation, corporate control, white knight, lockup fees, corporate governance

JEL: C72, D44, D82, G32, G34,

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1. Introduction

One of the main stylized facts of the US market for corporate control during the last decades is concerned with the dynamic of takeover processes.\(^1\) In particular, Betton et al. (2009) document for the period 1973-2002 the following trends: (i) hostile takeovers declined since the end of 1980s (from a peak in this decade), (ii) failed takeover decreased between sub-periods 1973-1989 and 1990-2002, and (iii) frequency of buyers’ previous stakes in the target (toeholds) decreased dramatically during the whole sample period.

As an attempt to account for these stylized facts, the present articles proposes a simple sequential bargaining model under complete information that characterizes conditions under which a takeover may be either friendly or hostile, or even unsuccessful. These conditions are related to the private benefits of control and selling incentives for the target management, the takeover premium, the toehold size, and the level of breakup fees. Furthermore, efficiency issues on takeovers are analyzed by characterizing the circumstances under which the target may end up in the buyer’s hands who doesn’t value it the most. Despite its simplicity, the model delivers testable implications that are, at first glance, consistent with the evidence above mentioned for the US market for corporate control.

Although previous theoretical works have also accounted for several of these US takeover trends in the last decades, they differ from our frame-\(^2\)

\[^1\] For some reviews on this evidence, see for instance, Eckbo (2009), Burkart and Panunzi (2008), Martynova and Renneboog (2008), Boone and Mulherin (2007b), Moeller et al. (2007), Andrade et al. (2001), and Holmstrom and Kaplan (2001).
work in various aspects. First, most of the received literature adopts an *auction*-based rather than a *bargaining*-based procedure to model a takeover process (Betton et al., 2009; Dasgupta and Tsui, 2003; Hansen, 2001; Bulow et al., 1999; Ettinger, 2009; Ravid and Spiegel, 1999; Burkart, 1995; Fishman, 1988). This is surprising as evidence shows that at least one half of the takeovers are conducted under a negotiation format (Aktas et al., 2010; Betton et al., 2008; Boone and Mulherin, 2007a). Second, in general all these takeover frameworks do *not* consider agency conflicts between shareholders and management, which implies to not take into account both private benefits of control and selling incentives for the management. Third, most contributions in this literature focus on explaining the motivation and dynamic of toeholds rather than the nature of the takeover outcome.

From all this previous literature, the closest theoretical model to ours is perhaps that developed by Betton et al. (2009). These authors also propose a takeover game that accounts for various elements present in the real-world market for corporate control. Besides the differences above mentioned, Betton et al. exhibit other two remarkable modeling discrepancies with our framework: (i) the definition of a hostile/friendly takeover, and (ii) the time-

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2 Among few exceptions we have Povel and Singh (2006) - who consider a hybrid procedure that combines a second-price auction with an exclusive deal -, and Loyola (2012b) - who compares auctions vs. negotiations in takeovers with toeholds.

3 Exceptions include Betton et al. (2009) - who however normalize to 1 selling incentives for the management -, and Goldman and Qian (2005) - who nevertheless only focus on hostile bid and do not consider the possibility of a friendly takeover.

4 Even the richest takeover setups proposed by Betton et al. (2009) and Goldman and Qian (2005) concentrate their conclusions on explaining the so-called *toehold puzzle*. 
ing of the game. Perhaps the first of these discrepancies is the principal explanation for both works to eventually obtain opposite results, and thus, it deserves a more-in-depth comment.

In Betton et al. (2009), what defines if a takeover is hostile or friendly is the rejecting/accepting response by the target management to a merger invitation made by a first bidder. Hence, if this invitation is rejected, the bidder launches an unsolicited (hostile) tender offer to shareholders, which is resisted by the target management. However, if this invitation is accepted, the process is classified as a friendly takeover even though the winning bidder finally removes any private benefits of control.

In contrast, in our model what defines the nature of a takeover is the initial intention a potential buyer has about removing/retaining the private benefits of the management. Accordingly, whenever a raider wants to remove these benefits once he takes over the company, the process is always classified as a hostile takeover, even though a provisional negotiation agreement takes place between the seller and this buyer. In this framework, a friendly takeover is then only carried out by a white knight that preserves the private benefits of control for the management.

As a consequence of this different definition of hostility in takeovers, the main results of both works also differ, even diametrically in some cases. For instance, whereas in Betton et al. (2009) a hostile takeover occurs when private benefits of control are sufficiently high, our model delivers exactly the contrary conclusion. In terms of efficiency, both works also yield different results: whereas in our framework the level of management private benefits is crucial to assign efficiently the target to a hostile raider who values it
more than a white knight, in Betton et al. these benefits play no role in that situation. A similar conclusion emerges by comparing both models regarding the possibility of an unsuccessful takeover: whereas in our work a failed takeover can be endogenously explained by the parameters of the model, in Betton et al. the probability of such an event is completely exogenous.

This paper proceeds as follows. Section 2 presents the baseline model, which describes a takeover negotiation game with private values and a complete information environment. Section 3 characterizes the conditions under which a takeover is eventually either friendly or hostile. Also, efficiency issues are examined. Section 4 performs extensions on two parameters of the baseline model (breakup fees and toeholds), which allows us to analyze conditions under which a takeover may eventually fail. Section 5 discusses the empirical implications from our theoretical model and its consistency with the main stylized facts of the US market for corporate control. Finally, Section 6 presents the principal conclusions and some possible extensions. All the proofs are collected in the Appendix.

2. A Takeover Negotiation Game

In this section we present the baseline setup adopted to model a takeover process. A company faces a takeover threat from two possible risk-neutral buyers: $A$ and $B$. The value of fully controlling the target to buyer $i$ is $v_i \in (0, 1]$; $i = A, B$. As we assume that this value is common knowledge, the setup here presented is one of complete information. We interpret $v_i$ as the private value that buyer $i$ assigns “to run the firm”, that is, a private synergy this buyer enjoys when taking over the target company and getting
absolute control of it.\(^5\)

The value that shareholders and outsiders assign initially to the target company is denoted by \(v_0\), which is common knowledge and normalized to zero. Thus, \(v_0\) represents the market value of the target under the current management, i.e., either at the pre-takeover time or after this process eventually fails.

We assume that the current management enjoys a private benefit of control given by \(\beta \in (0, 1]\). On the one hand, if buyer \(A\) takes over the company he removes such a benefit, and thus, we say that this is a takeover carried out by a hostile raider, or in short, a *hostile takeover*. On the other hand, if buyer \(B\) acquires the company this private benefit is not removed, and thus, we say that this process is a *friendly takeover* followed by a “white knight”.

The takeover is modeled as a sequential negotiation procedure, whose details are described below (see Subsection 2.1). This takeover procedure is conducted by the target management on behalf of all shareholders. In order to align incentives of both parties in this agency relationship, the management obtains a fraction \(\gamma \in (0, 1)\) of the net revenues coming from the target sale. Furthermore, it is assumed that there not exist coordination problems among shareholders.

An implicit but relevant assumption is that *all* shareholders *must* sell their stakes to the winning buyer (who *must* in turn buy all these stakes) following the pricing scheme stated by the rules of the negotiation process described below.

\(^5\)In the auction theory terminology, it is the so-called *private values* (PV) setting.
Lastly, it is assumed a zero discount rate.

2.1. Timing of the game

The takeover process is characterized by the following two-stage negotiation game.

**Stage I**

I.1. The target management makes a take-it-or-leave-it offer $p_A$ to buyer $A$ (the hostile raider), where $p_A$ is the price to be paid by buyer $i$ for each target share.

I.2. Buyer $A$ accepts or rejects this offer. If he rejects, the target company remains under the current ownership structure and management, and the game is over. Otherwise, if buyer $A$ accepts this deal, negotiation moves to the next round in which the management looks for a white knight that does not remove its private benefits $\beta$.

**Stage II**

II.1. The target management makes a new take-it-or-leave-it offer $p_B$ to buyer $B$ (the white knight).

II.2. Buyer $B$ accepts or rejects this offer. If he rejects, the target is sold to hostile raider $A$ according to the terms established in Stage I. Otherwise, the target company is sold to friendly buyer $B$, but the company must pay a per share breakup fee to raider $A$ given by $t \in (0, v_B)$.

The assumption on the role played by breakup fees builds on the so-called *commitment hypothesis* (Berkovitch and Khanna, 1990; Grossman and Hart, 1980). Accordingly, the main motivation for termination provisions in our model is protecting -and increasing the participation of- the first buyer in
the takeover negotiation process, in case of a deal previously attained with him be revoked by the management in a subsequent stage.

3. The Results

In this section we analyze three kind of results stemming from the takeover negotiation game: characterization of the equilibrium, possible outcomes, and efficiency issues.

3.1. The Equilibrium

The next proposition characterizes the subgame perfect Nash equilibrium (SPNE) of the dynamic game described in the previous section.

Proposition 1. The SPNE of the takeover negotiation game is characterized by:

Stage I

I.1

\[ p^*_A = \begin{cases} v_A + \varepsilon_A & \text{if } \frac{\theta}{\gamma} > \max \left\{ v_A, v_B - t + \frac{\theta}{\gamma} \right\} \\ v_A & \text{otherwise} \end{cases} \]

I.2

Buyer A

\begin{align*}
\text{accepts} & \quad \text{if } p^*_B = v_B + \varepsilon_B \text{ and } p_A \leq v_A \\
\text{or} & \quad \text{if } p^*_B = v_B \\
\text{rejects} & \quad \text{if } p^*_B = v_B + \varepsilon_B \text{ and } p_A > v_A
\end{align*}

Stage II

II.1

\[ p^*_B = \begin{cases} v_B + \varepsilon_B & \text{if } p_A > v_B - t + \frac{\theta}{\gamma} \\ v_B & \text{otherwise} \end{cases} \]
II.2

\[
\text{Buyer } B \begin{cases} 
\text{accepts} & \text{if } p_B \leq v_B \\
\text{rejects} & \text{otherwise}
\end{cases},
\]

where \( \varepsilon_A, \varepsilon_B > 0 \).

The equilibrium strategies resulting from this negotiation procedure deserve at least two comments. First, notice that the equilibrium premium \( p^*_i \) is so that the target management is able to extract all the buyers’ surplus when they accept their respective price offers. Of course, this result is the consequence of the first-mover advantage the management enjoys in this game.\(^6\)

Second, note that although \( \beta, \gamma \) and \( t \) do affect if target price offers are made to a buyer with the intention of getting from him either an accepting or rejecting response to the deal, these parameters do not affect the level of these offers.

3.2. Possible outcomes

From Proposition 1, it can be derived that in this corporate control game there are two possible outcomes: a friendly or a hostile takeover. These two cases result from the relationship among both buyers’ target values, the management’s private benefits and selling incentives, and breakup fees. This

\(^6\) Actually, evidence on the US market for corporate control suggests large gains for target shareholders and a neutral outcome for acquirers (Martynova and Renneboog, 2008; Kaplan and Stein, 1993; Kaplan et al., 2000; Betton et al., 2009; Eckbo, 2009). In that sense, high control premiums (an average around 45%) documented for the US seem to support our assumption of giving a strong bargaining power to the seller (for evidence on the percentage of these premiums, see for instance, Aktas et al., 2010; Betton et al., 2008; Boone and Mulherin, 2007b; Bates et al., 2006, Eckbo, 2009).
finding is formalized as follows.

**Corollary 1.** Let us define \( \bar{\beta} \equiv \gamma(v_A - v_B + t) \). The outcome of the takeover negotiation game has the following characteristics:

(i) The takeover is always successful.
(ii) If \( \beta \leq \bar{\beta} \), the company is sold to the hostile raider.
(iii) If \( \beta > \bar{\beta} \), the company is sold to the white knight.

Thus, the model proposed here points out that the nature of a successful takeover will depend on the level of the private benefits of control. Hence, if target management is low entrenched (\( \beta \leq \bar{\beta} \)), it will prefer to sell the company to the hostile raider. On the contrary, if the management is highly entrenched (\( \beta > \bar{\beta} \)), its optimal decision will be to assign the company to a white knight that preserves its high private benefits.

These results can be better understood if we perform some comparative statics over the threshold value \( \bar{\beta} \). If we define \( \Delta p^* \equiv v_A - v_B \) as the *takeover premium gap* when selling the company to a hostile instead of a friendly buyer, the threshold for private benefits can be rewritten as

\[
\bar{\beta} = \gamma(\Delta p^* + t). \tag{1}
\]

As from this definition it is clear that \( \bar{\beta} \) is increasing with \( \Delta p^* \), it follows that the management’s decision about the nature of the takeover (hostile or friendly) basically rests on the trade-off between: (i) its payoff coming from a (potential) higher premium (i.e. \( \Delta p^* > 0 \)) when selling to a hostile raider, and (ii) its preserved private benefits \( \beta \) when selling to a white knight.

It is important to stress that our condition for a hostile/friendly takeover is diametrically opposite to that characterized by Betton et al. (2009), which
is a consequence of the different definition of hostility that both works adopt. To understand this fact, let us consider the case of a highly entrenched target management who enjoys high private benefits of control. In the Betton et al.’s model the only way for the management to preserving these benefits with a positive probability is to reject the merger invitation, as in this case a white knight may win the auction run in a subsequent stage. According to the definition adopted by Betton et al., this target management’s rejecting response immediately transforms the process into a hostile takeover. In contrast, in our model the way for the management to preserving high private benefits is simply to set a sale price so that the white knight accepts to buy the company, as this class of buyer does not remove these benefits. According to our definition, and at variance with that adopted by Betton et al., this white knight’s accepting response immediately transforms the process into a friendly takeover.

Lastly, other elements that in our framework influence the nature of the takeover are the selling incentives $\gamma$, and the breakup fees $t$. As for these elements, from Corollary 1 and equation (1), it is straightforward to derive the next result.

**Corollary 2.** A hostile (resp. friendly) takeover is more (resp. less) likely as

(i) Breakup fees $t$ increase

(ii) Selling incentives $\gamma$ increase as long as $\Delta p^* > -t$.

A hostile takeover becomes, therefore, more likely as there are higher breakup fees saved from respecting an initial deal with a hostile raider (instead of attaining a new agreement with a white knight). This result is clearly contrary

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to the so-called entrenchment hypothesis, according to which termination provisions are a device for protecting management from hostile takeovers, and whose consequences are usually avoiding high-valued buyers and impairing allocational efficiency (Kahan and Klausner, 1996; Bulow and Klemperer, 1996; Schwartz, 1986). In contrast, our result is consistent with the commitment hypothesis (Berkovitch and Khanna, 1990; Grossman and Hart, 1980). According to this view, termination fees are a commitment device that compensates an initial raider if the target is finally acquired by a subsequent buyer, thus allowing the first buyer to visualize the end of the takeover negotiation process.

Corollary 2 also indicates that when the target premium gap $\Delta p^*$ is sufficiently high (not necessarily positive), more powerful selling incentives for the management (i.e. a higher $\gamma$) also increase the probability of a hostile takeover. The last situation may occur, for instance, if executive compensation packages include a payment based on stocks or stock options.

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7 A conclusion shared with the takeover auction model developed by Betton et al. (2009).

8 Evidence presented in Betton et al. (2009) for the US during the period 1973-2002 suggests that this premium gap is actually positive: in their sample the average final offer premium for friendly takeovers is 45.1%, and for hostile takeovers it is 60.9% (see Table 1, column 8). Also, Boone and Mulherin (2007b) show evidence that the nature of “unsolicited” of a takeover implies higher target returns in the US market. For continental Europe and UK, Martynova and Renneboog (2008) report various studies finding that stock returns for target shareholders are higher when facing a hostile rather than a friendly takeover.
3.3. Efficiency

We now examine whether the proposed model always yields efficient outcomes from a social viewpoint. In particular, we would like to know if one can guarantee that the target company will always end under the ownership structure that maximizes its value. Regarding this point, two cases deserve our attention.

First, notice that since it was assumed that \( v_i > 0 = v_0 \) for all \( i = A, B \), it is always efficient to sell the target company and not to keep it under the current ownership structure. Indeed, as Corollary 1 (Part i) points out, a successful takeover process is always guaranteed in the baseline model, and thus, this source of inefficiency can be ruled out.

Second, notice however that when \( v_A > v_B \), the negotiation procedure cannot ensure that always the firm will indeed be assigned to the hostile raider, which in this case, corresponds to the efficient outcome from a social welfare perspective (and also from the shareholders’ one). In fact, that result is not guaranteed in the context of our framework, as it is possible that the parameters of the model satisfy the following condition:

\[
v_B < v_A < v_B + \frac{\beta}{\gamma} - t.
\]

Rearranging the second inequality we obtain that

\[
\overline{\beta} < \beta,
\]

which, according to Corollary 1, is the condition that makes more profitable, from the management perspective, to sell the firm to the white knight than to sell it to the hostile raider.
Note that this result on efficiency contrasts with Betton et al. (2009), as in their auction takeover model management private benefits play no role over bid functions, and thereby, neither over the second-price auction rule used to assign the target company.9

4. Extensions

In this section we extend the baseline model in two directions: (i) by considering unrestricted breakup fees, and (ii) by including toeholds.

4.1. Unrestricted Breakup Fees

In the baseline model we assumed that per share breakup fees had an upper bound given by $v_B$. Let us explore what happens when this bound may no longer hold.

The next proposition characterizes the possible outcomes of the takeover game in such a situation.

**Corollary 3.** When $t$ is unrestricted, the takeover negotiation game has the following possible outcomes:

(i) If $t \leq v_B$ and $\beta \leq \beta$, the company is sold to the hostile raider.
(ii) If $t \leq v_B$ and $\beta > \beta$, the company is sold to the white knight.
(iii) If $t > v_B$, and $\beta \leq \gamma v_A$, the company is sold to the hostile raider.
(iv) If $t > v_B$, and $\beta > \gamma v_A$, the company remains under the current control

9In Betton et al. (2009) what indeed plays a crucial role on efficiency are the resistance costs suffered by the hostile bidder. As these costs give a competitive advantage to the white knight, the target can finally be inefficiently assigned to this friendly bidder even though he does not value it the most.
Thus, when breakup fees are not restricted, possible outcomes also consider an unsuccessful takeover. This occurs as long as both breakup fees and private benefits of control are sufficiently high (Case (iv) in Corollary 3). The intuition behind this result is as follows. When $t > v_B$, from the management viewpoint, to sell the firm to the white knight is always dominated by not to sell it at all, as the payoff of the second alternative is higher: $\beta > \beta + \gamma(v_B - t)$. This occurs even if the management is highly entrenched (a high $\beta$), as now $t$, the indemnification to be paid to the hostile raider, is so high that it even exceeds $v_B$, the revenues coming from the premium charged to the white knight. The relevant comparison for the management is thus between its payoff from selling to the hostile raider and its private benefits from keeping the control of the target, i.e., $\gamma v_A$ vs. $\beta$. Hence, the takeover will eventually fail whenever it holds the inequality $\beta > \gamma v_A$.\footnote{Taken jointly, both conditions of Case (iv) on $v_A$ and $v_B$ imply that a failed takeover is more likely when both buyers’ potential synergies from running the firm are relatively low.}

The last result has implications on the efficiency of the takeover process. In particular, the potential failure of the takeover introduces an additional source of inefficiency to that examined in Subsection 3.3. Since we have assumed that $v_i > 0 = v_0$ for all $i = A, B$, it is then always more efficient to sell the target company than not to sell it. Nevertheless, Case (iv) in Corollary 3 suggests that now a failure of the takeover cannot be ruled out, and thus, the selling negotiation procedure may inefficiently end up with the target in the current management’s hands.
To sum up, when breakup fees are unrestricted, they entail a trade-off related to the efficiency of the takeover outcome. Thus, if breakup fees increase, but until a threshold $v_B$ (i.e., the maximum premium charged to the white knight), the probability of an efficient hostile takeover increases as well, as these fees protect the hostile raider. Nevertheless, if breakup fees exceed such a threshold, the entire takeover process may eventually fail whether the private benefits for management are sufficiently high.

Notice that this unsuccessful takeover endogenously determined by the parameters of our model is clearly at variance with Betton et al. (2009), as in that work the probability of such an event is assumed to be completely exogenous.

4.2. Toeholds

Let us now assume that before the takeover process begins, hostile raider $A$ has a toehold, that is, a common-knowledge minority participation in the ownership of the target, and hence, in the selling surplus as well.\footnote{Evidence for the US shows that, although toeholds are nowadays quite rare, they are very frequent in hostile takeovers (see for instance Betton et al., 2009).} As $\theta \in (0, 1/2)$ denotes such a toehold, the term $(1 - \theta)$ then represents the fraction of the selling surplus going to nonbuying shareholders.\footnote{Buyer $B$, as a non-toeholder, plays then the role of an outside buyer.} Hence, we thus assume that the takeover negotiation process is now conducted by the target management only on behalf of such nonbuying shareholders.

As in the baseline model, an implicit assumption is that the selling process is a full takeover of the target. That is, all shareholders (in this case both buying and nonbuying ones) must sell their stakes to the winning buyer (and
he must buy these stakes), according to the pricing rules of the negotiation process described in Subsection 2.1.

In order to explore the role played by toeholds on the success/failure of the takeover, we assume, as in the previous subsection, that breakup fees $t$ are unrestricted.

The next statement characterizes the SPNE of our takeover negotiation game under the ownership structure above described.

**Proposition 2.** When the hostile raider has a toehold (i.e. $\theta > 0$), the SPNE of the takeover negotiation game is characterized by:

*Stage I*

$I.1$

$$p_A^* = \begin{cases} \frac{v_A}{1-\theta} + \epsilon_A & \text{if } \frac{\delta}{\gamma} > \max \left\{ v_A, v_B(1 - \theta) - t + \frac{\beta}{\gamma} \right\}, \\ \frac{v_A}{1-\theta} & \text{otherwise} \end{cases}$$

$I.2$

Buyer $A$

\[
\begin{cases}
\text{accepts} & \text{if } p_B^* = v_B + \epsilon_B \text{ and } p_A \leq \frac{v_A}{1-\theta} \\
& \text{or} \\
\text{rejects} & \text{if } p_B^* = v_B \\
& \text{if } p_B^* = v_B + \epsilon_B \text{ and } p_A > \frac{v_A}{1-\theta}
\end{cases}
\]

*Stage II*

$I.1$

$$p_B^* = \begin{cases} v_B + \epsilon_B & \text{if } p_A > v_B - \frac{t}{(1-\theta)} + \frac{\beta}{\gamma(1-\theta)} \\ v_B & \text{otherwise} \end{cases}$$
II.2

\[
B \begin{cases} 
\text{accepts} & \text{if } p_B \leq v_B \\
\text{rejects} & \text{otherwise}
\end{cases},
\]

where \( \varepsilon_A, \varepsilon_B > 0 \).

The takeover procedure is so that \( \frac{\partial p^*_A}{\partial \theta} > 0 \), which means that the management can get a higher target premium from buyer \( A \) as his toehold size increases. This result also implies that the selling procedure induces a discriminatory pricing policy against the toeholder, as the management imposes an extra-charge only in the price offer made to the hostile raider. To see that, note that if \( v_A = v_B = v \), the equilibrium accepting offers are so that:

\[
p^*_B = v < \frac{v}{1 - \theta} = p^*_A.
\]

This result confirms a finding from previous literature (Loyola, 2012a,b), in the sense that a price-enhancing strategy for the seller in a takeover with previous stakes in the target is to impose a bias against the strong buyer (in this case that with a positive toehold).

It is worthy to stress that the toehold size also plays a crucial role on the characteristics of the outcome resulting from our takeover game, as pointed out by the next two statements.

**Corollary 4.** Let us define \( \bar{t} \equiv (1 - \theta)v_B \), and \( \bar{\beta} \equiv \gamma [v_A - v_B(1 - \theta) + t] \).

When the hostile raider has a toehold, the takeover negotiation game has the following possible outcomes:

(i) If \( t \leq \bar{t} \) and \( \beta \leq \bar{\beta} \), the company is sold to the hostile raider.

(ii) If \( t \leq \bar{t} \) and \( \beta > \bar{\beta} \), the company is sold to the white knight.

(iii) If \( t > \bar{t} \), and \( \beta \leq \gamma v_A \), the company is sold to the hostile raider.
(iv) If \( t > \bar{t} \), and \( \beta > \gamma v_A \), the company remains under the current control and management.

As both cut-off values \( \bar{t} \) and \( \bar{\beta} \) depend on \( \theta \), the last corollary suggests that toeholds can affect not only the nature of the takeover (hostile or friendly), but also the success of the entire process. These properties are formalized as follows.

**Corollary 5.** *When the hostile raider has a toehold, it is verified that*

(i) A hostile (resp. friendly) takeover is more (resp. less) likely as toehold \( \theta \) increases.

(ii) A hostile (resp. friendly) takeover is more (resp. less) likely than when the hostile raider has no toehold at all.

(iii) A failed takeover is more likely as toehold \( \theta \) increases.

The last statement suggests that the acquisition of a previous stake in the target can be seen as a strategy of a potential buyer in order to increase his chance of success in a future hostile takeover attempt. As the proof of Corollary 5 indicates, this occurs because a buyer \( A \)'s toehold enlarges the region of private benefits from control for which the management eventually prefer selling the company to a hostile raider.\(^{13}\) Interestingly, this result is consistent with the evidence on previous target stakes in the US market for corporate control, for which, for instance, Betton et al. (2009) find that in

\(^{13}\) This result then posits a toehold-based advantage for a potential buyer in the context of a *negotiation*-based takeover, different from that attributed to toeholds in the context of an *auction*-based takeover, and related to either more aggressive bids (Burkart, 1995; Singh, 1998; Bulow et al., 1999) or an informational advantage (Povel and Sertsios, 2012).
hostile takeovers the presence of toeholds is much more frequent (50%) than in friendly takeovers (11%).

As for the efficiency issue, notice that previous target stakes held by the hostile buyer play a dual role. For instance, if we start from a situation in which \( t < \tilde{t} \), a sufficiently high increase of toehold \( \theta \) may lead to a sufficiently high decrease of \( \tilde{t} \) so that now \( t > \tilde{t} \). In that case, two countervailing effects come out. On the one side, as now the threshold for private benefits of control \( \beta \) increases from \( \tilde{\beta} \) to \( \gamma v_A \), the probability of an inefficient friendly takeover decreases. On the other side, however, if private benefits of control are high enough (i.e. \( \beta > \gamma v_A \)), one cannot rule out that the takeover process eventually fails, which in our model constitutes an inefficient outcome.

It worth stressing that this dual role of toeholds on efficiency is absent in the takeover model of Betton et al. (2009). On the one hand, when the hostile bidder’s toehold increases, an overbidding phenomenon on the part of this bidder further reduces the probability of an inefficient friendly takeover, which is consistent with our model. On the other hand, and in contrast to our framework, since in Betton et al. the probability of a no-bidder-wins scenario is exogenous, neither toeholds nor other parameters affect at all the success of a takeover.

5. Empirical Implications

The most general version of our model presented in Subsection 4.2, although simple, delivers some implications consistent with the evidence on the US market for corporate control during the 1980s and 1990s. First, our model suggests that hostile takeovers declined during the 1990s because ei-
ther the level of private benefits of control increased (the parameter $\beta$), or the critical threshold of this class of benefits decreased (the cut-off $\overline{\beta}$).

In the first case, we can reinterpret $\beta$ as an indirect measure of the quality of the corporate governance system. Accordingly, the framework here developed proposes that the fall of hostility in takeovers would be a sign that this system may indeed have worsened in the United States during the 1990s, as suggested by several accounting scandals at the early 2000s. In that sense, our model posits a hypothesis completely opposite to that of who claim that hostile takeovers declined because they were substituted by an improvement of the internal corporate governance systems, as the board of directors and the management of most of corporations would have adopted a shareholder value view during the 1990s (Holmstrom and Kaplan, 2001, 2003).

Alternatively, our model puts forward that this decline in hostility of takeovers may be explained by a decrease in the threshold $\overline{\beta}$. Specifically, this may have occurred due to a decrease in either: (i) the toehold size $\theta$ or its frequency, (ii) the premium gap $\Delta p^*$, (iii) the lockup fees $t$, or (iv) the power of incentives $\gamma$. Interestingly, corporate control transactions in the US deliver evidence consistent with a reduction in the first two parameters.$^{14}$ In fact,

$^{14}$On the side of lockup fees $t$, Boone and Mulherin (2007a) conclude that when correctly measuring termination provisions, and in contrast to previous empirical literature, there is little evidence that these provisions had increased in the US during the 1990’s. However, on the side of the power of incentives $\gamma$, two pieces of evidence suggest a contrary conclusion: (i) the sensitivity of top executives pay to shareholders returns increased tenfold between the 1980s and 1990s (Hall and Liebman, 1998), and (ii) the importance of equity-based compensation in managerial incentive schemes more than doubled during this period (Hall and Liebman, 1998; Hall and Murphy, 2002; Hall, 2003).
Betton et al. (2009) document that the percent of initial potential buyers with a toehold decreased dramatically from 22.2% during 1973-1989 to 7.5% during 1990-2002.\footnote{More recent evidence suggests that frequency of toeholds in takeovers conducted in the US and Canada was also very low (less than 2% of the sample) during the period 1998-2010 (Povel and Sertsios, 2012).} Also, according to evidence presented by Betton et al., the average offer premium decreased from 48.5% during 1973-1989 to 45.0% during 1990-2002. All this evidence thus suggests that our model provides some plausible explanations for the decline of hostile takeovers activity in the United States during the 1990s.

Second, our framework also points out that the reduction in the frequency of failed takeovers in the US during the 1990’s may be the consequence from either a decrease in the level of breakup fees (the parameter $t$) or an increase in the threshold of this class of fees (the cut-off $\tilde{t}$).\footnote{Betton et al. (2009) (Table 1, column 6) present evidence that the frequency of failed takeovers decreased from 36.1% during 1973-1989 to 26.4% during 1990-2002.} As recent evidence suggests that termination fees kept somewhat constant during the 1990’s (Boone and Mulherin, 2007a), one should focus on the cut-off $\tilde{t}$. In particular, our model predicts that an increase in this threshold is possible if the presence of toehold $\theta$ in takeovers decreases. As above discussed, evidence for the period 1973-2002 in the US market for corporate control confirms this prediction, as the frequency of initial stakes declined dramatically during this period (Betton et al., 2009).

Third, our analysis also sheds light on the stylized fact that hostile takeovers in continental Europe are less frequent than in the US (Burkart and
Panunzi, 2008; Tirole, 2006, p.43). As an illustration of this comparison, our setup suggests that this phenomenon can be explained by the fact that the incentive power $\gamma$ of managerial compensation packages in the United States is usually higher than in Europe, as it is indeed the case for stock-based compensation (see Tirole 2006, p. 21). Interestingly, evidence available for continental Europe in the 1990’s show an increase of both hostile takeovers activity and equity-based managerial compensation, which can also be considered as a prediction of our theoretical framework.

Lastly, the present model delivers a mixed prediction on two opposite factors which have influenced the dynamic of hostile takeovers in the US during the last decade. On the one side, there has been observed a decreasing importance of equity-based compensation in top managerial incentive schemes due to both various corporate scandals in the early 2000’s and some accounting changes in the mid-2000’s (Hall and Murphy, 2003; Hayes et al., 2012). In terms of our setup, this less importance of stock managerial compensation suggests a further reduction in the frequency of hostile takeovers, as this phenomenon can be interpreted as a decrease in $\gamma$ and, hence, in the cut-off $\bar{\eta}$. On the other side, there has been claimed a foreseeable improvement of corporate governance quality after the enactment of SOX in 2002. Related to this fact, our model suggests an increase of hostility in takeovers during the last decade, as a better corporate governance system can be associated to a reduction in the private benefits of control $\beta$. 

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6. Concluding Remarks

We have proposed a simple negotiation model that provides conditions under which a corporate control attempt may eventually result in a either friendly or hostile or unsuccessful takeover. In the baseline setup, in which breakup fees are restricted and there is no toehold, the takeover is always successful. Further, this takeover is finally hostile if the level of management’s entrenchment is sufficiently low, as in this case, the higher premium from selling the target to a hostile raider instead of a white knight more than compensates the removed private benefits of control. The entrenchment cut-off implicit in this result is so that a hostile takeover is more likely when increasing either the level of breakup fees or the power of selling incentives for the management.

In terms of efficiency, our baseline model suggests that one cannot rule out that the target may be inefficiently assigned to a white knight instead of a hostile raider, as there is a conflict of interests between shareholders and the management due to the presence of private benefits of control. This efficiency issue is more-in-depth characterized in a generalized version of our baseline model, in which: (i) breakup fees $t$ are unrestricted, and (ii) a hostile buyer’s toehold $\theta$ is allowed. In this more general framework, both parameters entail a trade-off on efficiency that now makes possible a failed takeover outcome.

Accordingly, when breakup fees are unrestricted, and they increase until some threshold (the maximum premium charged to the white knight), the probability of an efficient hostile takeover also increases, as these fees protect the hostile raider. However, if breakup fees exceed this threshold, the entire takeover process may eventually fail whether private benefits for the man-
agement are sufficiently high. Similarly, when a toehold for the hostile raider is allowed, it also has mixed implications for the efficiency of the process: whereas the probability of an inefficient friendly takeover is decreasing in the size of this target stake, a failed takeover is also possible whether toehold becomes too big.

Besides its consequences on efficiency, the presence of toeholds has also effects on the strategy and payoff of both selling shareholders and potential buyers, which can be summarized in two dimensions. First, when a toehold is allowed, the proposed negotiation model involves a selling mechanism with a timetable that not only gives priority to the hostile raider, but also charges a higher price to him. As a result, this discriminatory rule has a positive effect on the revenues of the target shareholders. Second, as in our model a toehold makes more likely a management’s decision in favor of a hostile raider, the acquisition of such previous target stakes can be interpreted as a potential buyer’s strategy to increasing his chance of success in a future hostile takeover attempt.

The model, although simple, accounts for two remarkable stylized facts in the US market for corporate control during the period between the 1970’s and 1990’s. In particular, it posits some testable hypotheses about the decline of hostile takeovers and the reduction of failed takeovers along this period. Interestingly, in our framework these hypotheses rest on elements highly present in the current debate on the market for corporate control such as conflicts of interests and private benefits of control, favored white knights, toeholds and breakup fees. Further, our analysis also shed lights on related issues such as the evolution of the corporate governance system in the United States during
the last decade, and the higher hostility of its takeover activity as compared to that observed in continental Europe.

It worth stressing that the bargaining model here proposed is realistic, as in practice we observe rounds of one-to-one negotiations in which the board of directors firstly attains a provisional agreement with a hostile raider. However, as the management wants to preserve its benefits of control, it is also frequent that this initial agreement be attained under the implicit threat of revoking it in a subsequent negotiation thanks to a white knight. As our model suggests, in these real-world negotiations breakup fees and toeholds play a crucial role on the takeover outcome.

There are several avenues to which the model here developed can be extended. First, resistance costs by the target management should be included, since the use of a wide array of anti-takeover defenses is prevalent in the market for corporate control, and whose presence can provide new insights into our understanding on the characteristics of the outcome of a takeover attempt. Second, it would be useful to perform a comparison of our bargaining-based takeover procedure with the auction-based takeover models mostly adopted by the game-theory literature on takeovers. This comparison can be made in terms of both optimality and efficiency, contributing in this way to the ongoing debate on the use of auctions vs. negotiations in real-world takeover processes. To do that, a third extension is however needed: the modeling of an asymmetric information environment rather than the complete information setup here adopted.
7. Appendix

Proof of Proposition 1. By applying backward induction, we proceed as follows.

Stage II.2

As buyer \( B \) faces a take-or-leave-it offer, it follows directly that at the last stage of the game the optimal decision rule for him is described by

\[
\text{Buyer } B \begin{cases} 
\text{accepts} & \text{if } p_B \leq v_B \\
\text{rejects} & \text{otherwise}
\end{cases}
\]

Stage II.1

Let \( U_M \) be the target management’s payoff. Then, at this stage the management offers a price \( p_B \) that considers the subsequent buyer \( B \)’s optimal strategy so that

\[
\max U_M = \begin{cases} 
\gamma p_A & \text{if } p_B > v_B \\
\gamma (p_B - t) + \beta & \text{if } p_B \leq v_B
\end{cases}
\]

from which it follows that

\[
p_B^* = \begin{cases} 
v_B + \varepsilon_B & \text{if } p_A > v_B - t + \frac{\beta}{\gamma} \\
v_B & \text{otherwise}
\end{cases}
\]

where \( \varepsilon_B > 0 \).

Stage I.2

At this stage buyer \( A \) decides about a take-it-or-leave-it offer, but also taking into account the optimal offer the management makes to buyer \( B \) in the next stage. Thus, he has to compare the payoff associated to three possible equilibrium paths: (i) \( v_A - p_A \) (if \( A \) accepts and \( B \) rejects), (ii) \( t \) (if
A accepts and B accepts), and (iii) zero payoff (if A rejects). Note that as by assumption \( t > 0 \), buyer A always accepts his offer whether B also accepts \textit{irrespective} of \( p_A \). However, if it is anticipated that B will reject his offer, A accepts or rejects his respective offer depending on the comparison \( v_A \) vs. \( p_A \). All this analysis is summarized by the following optimal decision rule for buyer A:

\[
\text{Buyer A} \begin{cases} 
accepts & \text{if } \ p_B^* = v_B + \varepsilon_B \text{ and } p_A \leq v_A \\
 & \text{or} \\
 & \text{if } \ p_B^* = v_B \\
rejects & \text{if } \ p_B^* = v_B + \varepsilon_B \text{ and } p_A > v_A
\end{cases},
\]
as by assumption \( t > 0 \).

**Stage I.1**

At this stage the target management compares the payoff stemming from three possible equilibrium paths yield in part by \( p_B^* \): (i) \( \beta \) (if A rejects), (ii) \( \gamma p_A \) (if A accepts and B rejects), and (iii) \( \gamma (p_B^* - t) + \beta \) (if A accepts and B accepts). Thus, the management offers a price \( p_A \) so that

\[
\max U_M = \begin{cases} 
\beta & \text{if } \ p_A > v_A \text{ and } p_B^* = v_B + \varepsilon_B \\
\gamma p_A & \text{if } \ p_A \leq v_A \text{ and } p_B^* = v_B + \varepsilon_B , \\
\gamma (p_B^* - t) + \beta & \text{if } \ p_B^* = v_B
\end{cases}
\]

and hence, it performs the following comparison

\[
\beta \text{ vs. } \max \{ \gamma p_A, \ \gamma (p_B^* - t) + \beta \} \Rightarrow \beta \text{ vs. } \max \{ \gamma v_A, \ \gamma (v_B - t) + \beta \}.
\]
The optimal price rule at this stage is thus given by

\[ p^*_A = \begin{cases} 
  v_A + \varepsilon_A & \text{if } \frac{\beta}{\gamma} > \max\left\{ v_A, v_B - t + \frac{\beta}{\gamma} \right\}, \\
  v_A & \text{if } \frac{\beta}{\gamma} \leq \max\left\{ v_A, v_B - t + \frac{\beta}{\gamma} \right\},
\end{cases} \]

where \( \varepsilon_A > 0 \).

The SPNE of this game consists then of all these equilibrium strategies.

\( \Box \)

**Proof of Corollary 1.** Based on the SPNE characterized in Proposition 1, and using the definition of \( \overline{\beta} \equiv \gamma(v_A - v_B + t) \) and the fact that \( t < v_B \), the following results can be shown.

(i) Notice that an equilibrium in which raider A rejects the management’s offer, and thus the takeover fails, is not possible. This is because the model’s parameters are so that one of the necessary conditions for A to reject, i.e. \( p^*_A = v_A + \varepsilon_A \), is never satisfied. In fact, from the optimal price rule at stage I.1 notice that this condition holds as long as \( \frac{\beta}{\gamma} > \max\left\{ v_A, v_B - t + \frac{\beta}{\gamma} \right\} \), or equivalently if \( \beta > \max\{\gamma v_A, \beta + \gamma v_A - \overline{\beta}\} \). It is possible to show that the last condition can never be verified as it requires that \( \overline{\beta} > \gamma v_A \), which is a contradiction since by assumption \( t < v_B \).

This then shows that the takeover is always successful, and thereby, we only now focus on the two equilibrium paths that are possible when \( p^*_A = v_A \).

(ii) It is easy to see that condition \( \beta \leq \overline{\beta} \) is equivalent to \( v_A \geq v_B - t + \frac{\beta}{\gamma} \). This inequality together with the fact that by assumption \( t < v_B \) imply that \( v_A \geq v_B - t + \frac{\beta}{\gamma} > \frac{\beta}{\gamma} \). According to Proposition 1, the last condition yields the following equilibrium path:

- \( p^*_A = v_A \).
- Buyer $A$ accepts,
- $p_B^* = v_B + \varepsilon_B$,
- Buyer $B$ rejects.

Hence, the company is sold to the hostile raider.

(iii) Alternatively, it is straightforward to check that condition $\beta > \beta$ and assumption $t < v_B$ imply jointly that $v_B - t + \frac{\beta}{\gamma} > \max\{v_A, \frac{\beta}{\gamma}\}$. Following Proposition 1, the equilibrium path is then in this case described by:
- $p_A^* = v_A$,
- Buyer $A$ accepts,
- $p_B^* = v_B$,
- Buyer $B$ accepts.

Hence, the company is sold to the white knight. \hfill \Box

**Proof of Corollary 2.** From the definition of $\beta$ given by equation (1), it follows that:

(i) $\frac{\partial p}{\partial t} = \gamma > 0$ since by assumption $\gamma \in (0, 1)$, and

(ii) $\frac{\partial \Delta p^*}{\partial \gamma} = \Delta p^* + t > 0$ as long as $\Delta p^* > -t$,

which in both cases, according to Corollary 1, implies a larger (smaller) region for values of $\beta$ satisfying the condition for a hostile (friendly) takeover. \Box

**Proof of Corollary 3.** Let us consider only the two additional cases when $t > v_B$, as cases (i) and (ii) when $t \leq v_B$ were already demonstrated in the proof of Corollary 1.

(iii) Condition $\beta \leq \gamma v_A$ and assumption $t > v_B$ imply that

$$\frac{\beta}{\gamma} + v_B - t < \frac{\beta}{\gamma} \leq v_A.$$
which according to Proposition 1 yields the following equilibrium path:
- $p_A^* = v_A$,
- Buyer $A$ accepts,
- $p_B^* = v_B + \varepsilon_B$,
- Buyer $B$ rejects.

Hence, the company is sold to the hostile raider.

(iv) Condition $\beta > \gamma v_A$ and assumption $t > v_B$ imply that
\[
\frac{\beta}{\gamma} > \max \left\{ v_B - t + \frac{\beta}{\gamma}, v_A \right\},
\]

Applying the last condition to Proposition 1, we have that $p_A^* = v_A + \varepsilon_A$.

However, this result itself does not ensure that buyer $A$ will necessarily reject its offer: this decision depends also on $p_B^*$. As for this decision, there are two alternatives for the target management: (a) $p_B^* = v_B + \varepsilon_B$ if $p_A^* > v_B - t + \frac{\beta}{\gamma}$, or (b) $p_B^* = v_B$ if $p_A^* \leq v_B - t + \frac{\beta}{\gamma}$. Since $p_A^* = v_A + \varepsilon_A$, these alternatives mean that the management can induce buyer $A$ to reject (accept) its offer indirectly through $p_B^*$ by means of a $\varepsilon_A$ sufficiently high (low).\(^{17}\) To establish which of both alternatives the management finally chooses, we compare its payoff coming from each one.

On the one hand, under alternative (a), $p_A^* = v_A + \varepsilon_A$ and $p_B^* = v_B + \varepsilon_B$ are jointly sufficient conditions for buyer $A$ to reject the management’s offer. Thus, the potential equilibrium path of this alternative implies that

\(^{17}\) The cut-off value is given by
\[
\varepsilon_A \equiv v_B - t + \frac{\beta}{\gamma} - v_A.
\]
the takeover fails. This outcome delivers a payoff for the management given by $U_M = \beta$.

On the other hand, under alternative (b), $p_B^* = v_B$ is a sufficient condition for both buyers to accept their respective management’s offers. Thus, the potential equilibrium path of this alternative is so that a friendly takeover takes place. This outcome delivers a payoff for the management given by $U_M = \beta + \gamma(v_B - t)$.

Since by assumption $t > v_B$, it is clear that the payoff for the target management is higher under alternative (a), and therefore, the equilibrium path associated to case (iv) in Corollary 3 results eventually in an unsuccessful takeover. \qed

**Proof of Proposition 2.** By applying the same line of reasoning that in the proof of Proposition 1, we proceed directly as follows.

**Stage II.2**

Buyer $B \begin{cases} \text{accepts} & \text{if } p_B \leq v_B \\ \text{rejects} & \text{otherwise} \end{cases}$.

**Stage II.1**

The target management offers a price $p_B$ so that

$$\max U_M = \begin{cases} \gamma p_A(1 - \theta) & \text{if } p_B > v_B \\ \gamma [p_B(1 - \theta) - t] + \beta & \text{if } p_B \leq v_B \end{cases},$$

from which it follows that

$$p_B^* = \begin{cases} v_B + \varepsilon_B & \text{if } p_A > v_B - \frac{t}{(1-\theta)} + \frac{\beta}{\gamma(1-\theta)} \\ v_B & \text{otherwise} \end{cases}.$$
where $\varepsilon_B > 0$.

**Stage I.2**

Buyer A

\[
\begin{align*}
\text{accepts} & \quad \begin{cases} 
  \text{if } p_B^* = v_B + \varepsilon_B \text{ and } p_A \leq \frac{v_A}{1-\gamma} \\
  \text{or} \quad \frac{v_A}{1-\gamma} \\
  \text{if } p_B^* = v_B 
\end{cases} \\
\text{rejects} & \quad \begin{cases} 
  \text{if } p_B^* = v_B + \varepsilon_B \text{ and } p_A > \frac{v_A}{1-\gamma}
\end{cases}
\end{align*}
\]

since $t + \theta v_B > 0$ as $t$, $\theta$ and $v_B$ are assumed to be strictly positive.

**Stage I.1**

The management offers a price $p_A$ so that

\[
\max U_M = \begin{cases} 
  \beta & \text{if } p_A > \frac{v_A}{1-\gamma} \text{ and } p_B^* = v_B + \varepsilon_B \\
  \gamma p_A (1 - \theta) & \text{if } p_A \leq \frac{v_A}{1-\gamma} \text{ and } p_B^* = v_B + \varepsilon_B \\
  \gamma [p_B^*(1 - \theta) - t] + \beta & \text{if } p_B^* = v_B
\end{cases}
\]

and hence, the target management carries out the following comparison

\[
\beta \text{ vs. } \max \{ \gamma p_A^*(1 - \theta), \gamma [p_B^*(1 - \theta) - t] + \beta \}
\]

\[
\Leftrightarrow \beta \text{ vs. } \max \{ \gamma v_A, \gamma [v_B(1 - \theta) - t] + \beta \}.
\]

Thus, it can be established that

\[
p_A^* = \begin{cases} 
  \frac{v_A}{1-\gamma} + \varepsilon_A & \text{if } \frac{\beta}{\gamma} > \max \left\{ v_A, v_B(1 - \theta) - t + \frac{\beta}{\gamma} \right\} \\
  \frac{v_A}{1-\gamma} & \text{otherwise}
\end{cases}
\]

where $\varepsilon_A > 0$.

The SPNE of this game is then characterized by considering all these equilibrium strategies.
Proof of Corollary 4. First, consider the case when $t \leq \overline{t}$. Following a similar analysis to that performed in the proof of Corollary 1, it is possible to show that when buyer $A$ has a toehold, $\overline{\beta}$ constitutes the new threshold for the private benefits of control $\beta$ under (over) which a hostile (friendly) takeover materializes. This demonstrates parts (i) and (ii). Second, consider now the case when $t > \overline{t}$. Applying the same line of reasoning of proof of Corollary 3, it is possible to show that the relationship $\beta$ vs. $\gamma v_A$ plays the same pivotal role in Proposition 2 than in Proposition 1, which proves parts (iii) and (iv). \hfill \Box

Proof of Corollary 5. (i) To perform this proof we fix an arbitrary value $t \leq \overline{t}$, as otherwise toehold $\theta$ does not affect the relevant cut-off $\gamma v_A$ for a hostile takeover. From the definition of $\overline{t}$, it follows that $\overline{\frac{\partial \overline{\beta}}{\partial \overline{\theta}}} = -v_B < 0$ as by assumption $v_B > 0$, which means that we have to analyze only two relevant cases after an increase of $\overline{\theta}$: (a) despite a decrease in $\overline{t}$, it still holds true that $t \leq \overline{t}$, and (b) because a decrease in $\overline{t}$, it now holds that $t > \overline{t}$. First, in case (a), notice that $\overline{\frac{\partial \overline{\beta}}{\partial \overline{\theta}}} = \gamma v_B > 0$ since by assumption $\gamma, v_B > 0$. According to the role played by $\overline{\beta}$ in the outcome of the takeover game (see Corollary 4, parts (i) and (ii)), the last result implies directly statement (i) in Corollary 5. Second, in case (b), if $\overline{t}$ decreases from $\overline{t}_0$ to $\overline{t}_1$ because an increase of $\overline{\theta}$, it is verified that

$$\overline{\beta} \equiv \gamma v_A + \gamma (t - \overline{t}_0) \leq \gamma v_A.$$ 

The last inequality means that the new cut-off of private benefits of control for a hostile takeover to occur is higher, and thus, this class of takeover is in case (b) more likely as well.
(ii) Let us start from a situation in which there is no toehold and we fix an arbitrary value $t \leq v_B$. We analyze only two relevant cases in order to compare this initial situation with a new one in which $\theta$ is positive: (a) it holds true that $t \leq \bar{t} < v_B$, and (b) it is now verified that $\bar{t} < t \leq v_B$. First, in case (a), notice that \( \bar{\beta} \equiv \beta + \gamma v_B \theta > \beta \) since $\theta > 0$. According to the role played by $\bar{\beta}$ in the outcome of the takeover game (see Corollary 4, parts (i) and (ii)), the last result implies immediately Part (ii) in Corollary 5. Second, in case (b), it is verified that

\[
\bar{\beta} \equiv \gamma v_A + \gamma(t - v_B) \leq \gamma v_A.
\]

This inequality implies that the new threshold $\gamma v_A$ for $\beta$ until to which a hostile takeover takes place when $\theta > 0$ (see Part (iii) in Corollary 4) is higher than the similar threshold $\beta$ when there is no toehold at all. As a result, one can conclude that a hostile takeover is also more likely in case (b).

(iii) It follows directly from the fact that \( \frac{\partial A}{\partial \theta} < 0 \) and the role played by $\bar{t}$ in the success of the takeover (see Corollary 4, Part (iv)).

\[
\square
\]

References


Loyola, G., 2012b. Auctions vs. negotiations in takeovers with initial stakes. Finance Research Letters 9, 111-120.


