Abstract
The recent financial crisis showed once again the dangers of having high levels of short term debt. In this paper we show that international investors prefer to finance long term projects with short term debt – that bears risk of self fulfilling crises. Short term debt enlarges their menu of assets allowing them to extract surplus from domestic entrepreneurs. The risk of a self fulfilling crisis acts as an entry barrier preventing other funds from flowing into the country. Lenders can assure this equilibrium through some coordination device e.g. a hedge fund.

Keywords: short term debt, financial crises, coordination.

JEL codes: F30, F32, F34, G15.

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Xavier Ordeñana
ESPAE Graduate School of Management
Escuela Superior Politécnica del Litoral (ESPOL)
Malecón 100 and Loja
Guayaquil, Ecuador
Telf.: +593 (4) 2530383
Fax: +593 (4) 2530057
Contact: xordenan@espol.edu.ec

Gustavo Solórzano Andrade
Centro de Investigaciones Económicas
Facultad de Economía y Negocios
Escuela Superior Politécnica del Litoral
Km 30.5 Vía Perimetral
Telf: (+593) 4 2260 097
Contact: gsolorza@espol.edu.ec
1 Introduction

A particular feature of emerging market crises is the excessive amount of short term borrowing\(^2\). On the other hand, Aizenman and Hutchison\(^4\) show how countries with less balance sheet exposure i.e. lower levels of short term debt resist better to the recent global financial crisis. Similar results were found by the International Monetary Fund in a recent project \(^\[13\]\).

During the last fifteen years, there has been some research in this area trying to explain why this kind of borrowing is dangerous. More recently, there has been some work seeking to explain the rationale behind the existence of short term debt. That is, if short term debt is bad, why do we observe it? In this paper, we will present a possible explanation to answer this question. When countries borrow short term, they are exposed to the risk of a self fulfilling crisis\(^3\). When creditors believe countries will not be able to honor their debt, they do not roll over the loans, and then countries effectively cannot pay. Hence, bad expectations may produce a crisis. The literature has concentrated in explaining why short term debt is "bad", but not on why does short term debt exist in the first place\(^4\).

Other authors have focused on the implementation of policies that rule out the possibility of these crises, going from regulation of capital flows to the proposal of a new set of contracts. In a survey by Rogoff \(^5\), he analyzes the potential role of international institutions, such as an international lender of last resort or an international bankruptcy court. He also shows the effect of controls on capital flows (both inflows and outflows). However, he mainly concentrates on how to avoid the crises given the debt structure and does not question the debt structure itself.

It should be clear that countries are not interested in taking self fulfilling crisis risk, mainly because of the losses observed when there is such a crisis, but also because the high interest rates that are needed to attract funds. Thus, it is surprising that we observe short term borrowing given the monopoly power any debtor has on her debt structure and therefore on her risk.

This paper is aimed on explaining this particular feature of the financial structure. In particular, given that debtors are negatively affected by short term debt, the question we want to address is "Is it in the interest of creditors that countries finance long term projects with short term debt? If so, how do they accomplish it?"

In order to answer these questions, we model a small country with long term investment projects to be financed by specialized risk averse foreign investors. The economy produces many non tradable intermediate goods that are in turn used to produce a tradable final good. The technology of the final good is crucial. We assume that there is a critical mass of inputs required to have positive production of the final good. This assumption creates a linkage between intermediate firms that will in turn affect the optimal debt structure that firms take.

If this project is financed with long term debt (i.e. if the project and the debt have the same maturity), there is no risk\(^5\). This together with the abundance of foreign resources competing on

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\(^2\) See, for example Krugman \([10]\).

\(^3\) See for instance Cole and Kehoe \([6]\) and Chang and Velasco \([12]\). Also, Rodrik and Velasco \([2]\) show empirically the importance of short term debt in predicting crises.

\(^4\) There are few exceptions. See, for example, Broner et al \([3]\).

\(^5\) In other words, we are assuming there is no production risk. This assumption can be easily removed.
the project, imply zero premium for investors, i.e. they will get the world risk free rate. Thus, in this scenario all the surplus of the project goes to the entrepreneurs.

But, the project can also be financed with short term debt (that is debt maturity shorter than the project: there is maturity mismatch). The existence of production linkages eliminates the monopoly power of the borrowers on their risk. If a large fraction of firms borrow short term, the possibility of a crisis on those firms will produce a collapse of the final good sector reducing the demand for all intermediates. Thus, even when borrowing fully in long term, a firm can be risky. The liquidity risk acts like an entry barrier limiting the amount of resources competing for the project. This allows international investors to extract part of the surplus (otherwise taken by entrepreneurs). Then, the investor’s portfolio is composed by both risk free investment in the world’s safe technology (the country is small and does not exhaust resources) and risky investment in the country.

This paper will argue that this last portfolio (composed partly by short term debt) is preferred by the lenders to the one composed by long term debt. Notice however that this does not imply that creditors like crises; they just like the possibility of it. On the other hand, entrepreneurs prefer to avoid maturity mismatches. However, high enough term premium\(^6\) can induce entrepreneurs to finance their project with short term debt. In other words, an equilibrium with a project fully financed by short term debt can exist.

Which equilibrium would be observed? We will argue that if, when deciding the investment creditors can coordinate, they will choose to finance these projects with short term debt. This coordination can be done through some international financial intermediary, e.g. a hedge fund. However, coordination is not perfect. The existence of a hedge fund is not enough to avoid the possibility of a crises: investors can run on the hedge fund, forcing the hedge fund to run on the country.

The paper is organized as follows: Section 2 presents a model of debt maturity choice in a competitive bond market. Section 3 does welfare comparisons between equilibria for both agents and for the economy as a whole. Section 4 introduces a financial intermediary (a hedge-fund) to obtain a unique equilibrium. Section 5 concludes.

\(^6\) We refer to term premium as the difference between long term and short term interest rate.
2 A Model of Optimal Debt Structure in a Competitive Bond Market

Let the economy be composed by a continuum of firms that produce intermediate goods to be sold to a final sector. The intermediate goods are differentiated and non-traded whilst the final good is traded. In addition, there is a continuum of international lenders that behave perfectly-competitive. All lenders and entrepreneurs live for three periods: 1, 2, 3.

Each entrepreneur maximizes her utility function \( E\{U(\cdot)\} \). She has an illiquid investment project that lasts two periods. She invests an amount \( K_i \) in period 1 and the project yields \( f(K_i) \) in period 3. The production function \( f \) is a continuous, concave, twice-differentiable function that satisfies Inada conditions. We assume that in period 3, once debts are paid, she consumes. If, for any reason at period 2, she has to disinvest part of the capital, she will only get a fraction \( \varphi \) of it. The remaining fraction is destroyed due to costly disinvestment.

On the other hand, the representative lender is a risk averse investor with wealth \( W \). She can invest in a risk-free storage technology that we assume pays 1, or lend in two maturities: short term loans \( (D) \) which last one period and long term loans \( (K - D) \) which last two periods (charging respectively net rates \( r_S \) and \( r_L \)). If she decides to lend short term, at period 1 she will have another decision to take: She will decide whether to roll over the debt or to use these resources to invest in the risk-free technology. Finally, in period 2 she consumes. Thus, the lender’s problem is to maximize her expected utility \( E\{V(\cdot)\} \), where \( V \) satisfies all desirable properties.

The final sector uses different inputs denoted by \( Z_i \). Moreover, the final sector can only produce if there is a sufficiently large amount of inputs available.

Formally, let \( N = \{i : Z_i > 0\} \), then the final good production is

\[
Y = \begin{cases} 
\frac{1}{\alpha} \left[ \int_0^1 Z_i^\alpha di \right]^{\frac{1}{\alpha}} & , \ m(N) \geq \mu \\
0 & , \ m(N) < \mu 
\end{cases}
\]  

(1)

Where \( \mu \) is the minimum amount of inputs required to have positive production. This creates a linkage between firms to insure that in case of financial distress, all firms are affected.

**Definition 1** Let \( \bar{K} \) be the level of capital for which \( f'(K) = 1 \).
The amount of capital invested in the country will never exceed $\bar{K}$. This would be the capital invested if there were no issues of liquidity. For any level of capital higher than this threshold, the return would be lower than the risk-free technology.

**Assumption 1** *Available resources for entrepreneurs are not scarce, i.e. $\bar{K} < W$.*

This assumption will imply that competition among entrepreneurs will not exhaust surplus.

**Assumption 2** *International investors are specialized: they invest a non negligible fraction of their portfolio in the country.*

Notice that lenders are risk averse. If lenders invest a negligible fraction of their wealth in a country, they would behave as risk neutral. The specialized investors assumption prevents this from happening.

The timing of the problem is as follows

- At $T = 1$, lenders and entrepreneurs decide the amount to be invested and the debt structure.
- At $T = 2$, lenders holding short term debt decide to rollover or not their debt.
- At $T = 3$, entrepreneurs sell their intermediate good to the final output sector and debt is repaid.

**Definition 2** *(Competitive Bond Market Equilibrium)* The equilibrium will be characterized by a set of prices $\{P_i, r_{Si}, r_{Li}\}$ $\forall i$, and quantities $\{D_i, K_i\}$ $\forall i$, such that:

(i) Given $\{P_i\}$, final sector firms maximize profits.
(ii) Given $\{r_{Si}, r_{Li}\}$, lenders maximize utility.
(iii) Given $\{r_{Si}, r_{Li}\}$ and final sector demand, entrepreneurs maximize utility.
(iv) Markets clear.
2.1 Final Good Sector

At $T = 3$, the final good sector firms buy inputs to produce the only tradable good in the economy ($Y$). We will use the price of the final good as the numeraire, i.e. $P_Y = 1$. The problem of the final good sector is

$$\max_{\{Z_i\}} \left\{ Y - \int_0^1 P_i Z_i \, di \right\}$$

where $P_i$ is the price of intermediate $i$, and $Y$ is the production as defined in Equation 1.

The first order conditions are:

$$Z_i : \quad \frac{1}{\alpha} \left[ \int_0^1 Z_j^\alpha \, dj \right]^{\frac{1-\alpha}{\alpha}} Z_i^{\alpha-1} - P_i = 0; \quad \forall i$$

Let $G_i$ be the revenue function of intermediate $i$, i.e. $G_i = P_i Z_i$. The first order condition implies$^6$:

$$G_i = \frac{1}{\alpha} \left[ \int_0^1 Z_j^\alpha \, dj \right]^{\frac{1-\alpha}{\alpha}} Z_i^\alpha$$

2.2 Roll Over Decision

At $T = 2$, lenders have to decide if they roll over the debt or not.

Let us begin the analysis by analyzing the possibility of a run on a single firm. Denote $\beta_i$, as the fraction of lenders that at period 1 decide to run on firm $i$. Again, we refer to "run" as deciding not to renew their short term contracts from period 1 to period 2.

If the revenue of firm $i$ after paying the long term bond holders, is still capable of paying the short term bond holders that did not run $(1 - \beta_i)$, then a run was definitely not optimal: The $\beta_i$ investors that ran should not have run, i.e. $\beta_i = 0$.$^7$

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$^6$This expression is valid whenever production of intermediate is completed. In some cases, this will not be the case. If production is not completed, $G_i$ would be equal to the residual value of the firm. We leave this explanation for later.

$^7$Actually, they are indifferent since from period 1 to period 2, the project yields as the safe technology. However, we assume that there is some small cost of running.
On the other hand, suppose that firm $i$ is not capable of repaying the $1 - \beta_i$ bond holders that did not run. In this case, a run was optimal, and all investors should have done it, $(\beta_i = 1)$. Lemma 1 states this formally$^8$.

**Lemma 1** For each firm $i$, there cannot be partial runs: either all investors run or none of them do. In particular

$$G(K_i - \beta_i(1 + r_{Si})(K_i - D_i) - (1 + r_{Li})D_i) \geq (1 - \beta_i)(1 + r_{Si})D_i \implies \beta_i = 0$$

$$G(K_i - \beta_i(1 + r_{Si})(K_i - D_i) - (1 + r_{Li})D_i) < (1 - \beta_i)(1 + r_{Si})D_i \implies \beta_i = 1$$

Let us now consider the possibility of runs on different firms. Lemma 1 implies that each firm is either fully attacked or is not attacked at all. It is easy to see that for some levels of short term debt,

$$G(K_i - (1 + r_{Si})D_i/\varphi) - (1 + r_{Li})(K_i - D_i) \geq 0 \quad (2)$$

i.e. firm $i$ is able to pay its long term bond holders even in the case of a full run on its short term obligations. Clearly, for levels of debt that satisfy 2, firm $i$ will not be attacked$^9$.

On the other hand, whenever

$$G(K_i) < (1 + r_{Si})D_i + (1 + r_{Li})(K_i - D_i), \quad (3)$$

firm $i$ will be fully attacked. Lemma 2 states these two results formally.

**Lemma 2** For each firm $i$,

- $G_i(K_i - (1 + r_{Si})D_i/\varphi) - (1 + r_{Li})(K_i - D_i) \geq 0 \implies Prob(\beta_i = 1) = 0$
- $G_i(K_i) < (1 + r_{Si})D_i + (1 + r_{Li})(K_i - D_i) \implies Prob(\beta_i = 1) = 1$

For further notation, we will refer to $D_i^\ast$ as the level of short term debt that satisfies 2 with equality, i.e. as the maximum level of short term debt for which firm $i$ will be attacked with probability zero$^{10}$.

$^8$Notice that the short term interest rate from period 1 to period 2 is always zero since uncertainty is fully resolved in period 1

$^9$Notice that $G()$ depends on prices. Prices can be low enough so that Equation 2 is never satisfied

$^{10}$Clearly $D_i^\ast$ depends on the demand of the final good sector.
Note that Lemma 2 does not cover all possible levels of short term debt. What can we say about these other levels of short term debt? Entrepreneurs with these levels of short term debt will have positive probability of runs. We will assume for these firms that the attack decision follows a sunspot.

Let $B \subseteq [0, 1]$ be the set of firms that are attacked, i.e. $i \in B$ only if $\beta_i = 1$. Furthermore, denote $S$ to be the set of firms that for any possible "attack" set $B$, satisfy 2. In other words, $S = \{i : \forall B, D_i \leq D^*_i\}$. Finally, denote $R = [0, 1] - S$ as the set of firms that are not in $S$.

Notice that a possible attack can be anything going from the empty set to the set $R^{11}$. Moreover, if a firm $i$ is not part of an attack $B$, but for this event firm $i$ satisfies 3, then $B$ should have probability zero. In order to satisfy these conditions we will take a simple and often-used distribution focusing on attack to all firms or no attack at all.

**Assumption 3** Lenders will decide to run or not following a sunspot: With probability $\pi$ lenders will attack all firms, and with probability $1 - \pi$ they will not attack at all.

$$\text{Prob}(B = \emptyset) = 1 - \pi$$

$$\text{Prob}(B = R) = \pi$$

This is not the only distribution consistent with lemma 2.

### 2.3 Optimal Debt Choice

We are now capable of analyzing the problem of lenders and entrepreneurs. Let us begin by defining $r_i$ as the weighted average return rate of investment in firm $i$, i.e.

**Definition 3** Let’s define the average interest rate for entrepreneur $i$ as the weighted average of the short term and long term interest respectively.

$$r_i = \frac{D_i}{K_i} r_{Si} + \frac{K_i - D_i}{K_i} r_{Li}$$

\(^{11}\text{It should be clear to see that }B \cap S \neq \emptyset \text{ implies } \text{Prob}(B)=0.\)
Moreover let \( R_1 \) and \( R_2 \) be a partition of the set \( R \) \((R_1 \cup R_2 = R \) and \( R_1 \cap R_2 = \emptyset) . \) \( R_1 \) will be composed of firms such that \( i \in R_1 \) if and only if \( D_i^* < D_i \leq \varphi K . \) Naturally, \( i \in R_2 \) if and only if \( D_i \geq \varphi K . \)

Finally denote \( X_{S_i} \) (\( X_{L_i} \)) as the amount of debt invested short (long) term in firm \( i \).

**Lenders’ Problem**

Lenders will solve the following problem:

\[
\max_{(x_S,x_L)} \left\{ (1 - \pi) V \left( W + \int_0^1 x_S r_S + x_L r_L \right) + \pi V \left( W + \int_S [x_S r_S + x_L r_L] + \int_{R_1} x_S r_S + x_L \left( \frac{G_i(K_i - (1 + r_S)D_i/\varphi)}{K_i - D_i} - 1 \right) \right) \right\}
\]

It is easy to show that the first order conditions of this problem imply that lenders will charge the following interest rates:

\[
\forall i \in S, \quad r_i = 0 \quad (4)
\]

\[
\forall i \in R_1, \quad r_i = \frac{\pi}{1 - \pi} \frac{V'(C_R)}{V'(C_{\emptyset})} \left[ K_i - D_i - G(K_i) - (1 + r_i)K_i \right] \quad (5)
\]

\[
\forall i \in R_2, \quad r_i = \frac{\pi}{1 - \pi} \frac{V'(C_R)}{V'(C_{\emptyset})} (1 - \varphi) \quad (6)
\]

Where \( C_B \) is the payoff for the lenders when the event \( B \in \{\emptyset, R\} \) occurs.

**Entrepreneurs’ Problem**

What is the optimal response of entrepreneurs? Notice entrepreneurs act as monopolistic competitive in their own intermediate good. They maximize the following problem

\[
\max_{K_i,D_i} \left\{ (1 - \pi) U(G(K_i) - (1 + r_i)K_i) + \pi U(I_{i(S)} (G(K_i) - (1 + r_i)K_i)) \right\}
\]

\[
\text{s.t. Equations (4) to (6)}. \]

\( I_{i(S)} \) is an index function that takes the value of one whenever \( i \in S \).

Whenever feasible, entrepreneurs will choose to be in \( S \). Symmetry of the problem implies that either all entrepreneurs are in \( S \) or none of them are.
Suppose first that \( \forall i \in S \), i.e. every entrepreneur satisfies Equation 2 (with \( r_S = r_L = 0 \)). If this is the case, all entrepreneurs will produce a positive amount of intermediates, \( (Z_i > 0) \), and thus, there is positive production (see Equation 1).

Moreover, the entrepreneur’s problem maximization and symmetry imply

\[
G'(K) = f'(K) = 1
\]

i.e. \( K = \bar{K} \).

Furthermore, notice that \( G_i = \frac{1}{\alpha} f(\bar{K}) \), \( \forall i \). It should be easy to see that there is a positive level of short term debt \( D^* \) such that Equation 2 is satisfied. So, \( S \) was indeed feasible, therefore they choose to be in \( S \) and we have an equilibrium.

On the other hand, suppose that \( S \) is not feasible. Moreover assume \( m(R_2) > 1 - \mu \), where \( m(R_2) \) denotes the measure of the set \( R_2 \). In this case, whenever \( B = R \), then \( Y = 0 \). Thus, the revenue function is \( G_i = \varphi K_i - D_i \) (in other words, the residual value). So, \( S \) is indeed not feasible and every entrepreneur \( i \) is indifferent about her debt structure\(^{12} \). The first order condition of the problem now imply

\[
K_i : \left[ \int_0^1 Z_j^\alpha d\mu \right]^{1-\alpha} Z_i^{\alpha-1} f'(K_i) - (1 + r_i) = 0
\]

By symmetry among entrepreneurs, we have: \( Z_i = Z_j \). This condition and the lenders supply of funds give the solution for the debt market:

\[
r = \frac{\pi}{1 - \pi} \frac{V'(W - (1 - \varphi)K)}{V'(W + rK)} (1 - \varphi)
\]

\[
G'(K) = 1 + r
\]

Equations 7 and 8 define the second equilibrium in the economy. Notice there is an investment level lower than \( \bar{K} \).

Proposition 1 There are two equilibria in the model:

- \( m(S) = 1 \), \( D \leq D^* \)
- \( m(R_2) > 1 - \mu \), \( D \geq \varphi K \)

\(^{12}\)To see that substitute \( G_i = \varphi K_i - D_i \) in Equation 5 and notice it becomes identical to 6
Proposition 1 shows the two equilibria in the model. In the chart below we characterized the respective interest rates, utility levels, debt amounts and investment amounts in each of the two equilibria. In the Appendix, we show that there are no other equilibrium.

### Characterization of Equilibria

#### Equilibrium E0

\[
\begin{align*}
    r &= r_S = r_L = 0 \\
    f'(K) &= 1 \\
    D &\leq D^* \\
    U(G(K) - K) &- V(W) \\
\end{align*}
\]

#### Equilibrium E1

\[
\begin{align*}
    r &= \frac{\pi}{1 - \pi} \frac{V'(W - (1 - \phi)K)}{V'(W + rK)} (1 - \phi) \\
    r_S &= \frac{r}{1 - \phi} \left(1 - \frac{K}{D}\right) \\
    r_L &= \frac{r}{1 - \phi} \\
    f'(K) &= 1 + r \\
    D &\geq \phi K \\
    (1 - \pi)U(G(K) - (1 + r)K) &- (1 - \pi)V(W + rK) + \pi V(W - (1 - \phi)K) \\
\end{align*}
\]

3 Equilibria and Welfare Comparisons

We have seen that when deciding the maturity structure, there exists multiple equilibria: the project can be financed with long term debt (E0) or with short term debt (E1)\(^{13}\). We now turn to answer the question: what equilibrium does each agent prefer? It should be obvious to see that entrepreneurs prefer the equilibrium with long term debt (E0). The project will be completed with probability one, it will generate resources, interest rates are zero and there is

\(^{13}\)Remember we have assumed without loss of generality that \(D = 0\) in \(E0\) and \(D = K\) in \(E1\)
But, what do lenders prefer? One can think that they should also prefer equilibrium $E_0$. There is no possibility of crisis, and investors always seek to avoid crises. Don’t they? Let us compare the perceived utility in both equilibria. In equilibrium $E_0$, when they invest only in long term debt, lenders have only one investment opportunity: both long term debt and the risk free technology pay 1. On the other hand, when they finance the project investing in short term debt, lenders are investing in two assets: Short term debt ($K$) and risk free technology ($W - K$). But notice, that they can always replicate their investment in $E_0$, by fully investing their wealth in the risk free technology: they optimally choose not to. Therefore, their utility must be higher in $E_1$.

In particular, the perceived utility of the lenders in the case of long term financing ($E_0$) is $V_0 = V(W)$ while in the case of $E1$ is $V_1 = (1 - \pi)V(W + rK) + \pi V(W - (1 - \varphi)K)$. It should be clear that $V_1 > V_0$: otherwise they could always set $K = 0$ and thus $V_1 = V_0$. Proposition 2 states this point.

**Proposition 2** Lenders prefer to finance the investment projects with short term debt. In other words, they prefer Equilibrium $E_1$ to Equilibrium $E0$.

**Proof.**
See Appendix. □

Figure 1 gives an intuition for this result. We plot an expected return - risk graph and compare the utility in both equilibria. $V_0$ denotes the utility perceived by lenders when investing long term. Clearly, this type of debt bears no risk and thus the equilibrium point lies on the vertical axis. When one includes short term debt (with a high return and high risk), the average return will be the point marked as return with short term debt. Then, $V_1$ would be the utility of the lenders when investing $K$ in short term and $W - K$ in the safe technology. It is easy to see that $V_1 > V_0$.

Why are lenders better off in an equilibrium where crises are possible? Notice that in equilibrium $E0$, given that $W > K$ (i.e. resources are not scarce) lenders cannot extract any surplus from the entrepreneurs. Why? If lenders wish to charge more than 1 on the debt, the extra $W - K$ will immediately flow to the project and thus, the equilibrium rates will again be the risk free ones.
On the other hand, when they finance the project with short term debt, lenders have optimally chosen to place $K$ in the project and $W - K$ in the risk free technology, although the latter pays strictly less than the project. Why don’t these resources flow into the project? Clearly, the reason is that the project now bears risk. Thus, the risk of a crisis is acting as an "entry barrier" preventing other lenders from placing their resources in the entrepreneurs risky project. This allows lenders to extract surplus from the entrepreneur making this equilibrium more attractive.

It is important to notice that lenders prefer the equilibrium debt level where a crisis is possible ex ante, but clearly they want the crisis not to happen ex post.

Figure 2 shows another interpretation of this result. The line $f'(k)$ is the demand of funds of entrepreneurs and the lines $ST$ and $LT$ are the supply of
funds by the lenders. As in a basic supply and demand model, we can calculate and compare surplus from each agent. In the graph on the left, the equilibrium when the project is financed with long term debt, all the surplus goes to the entrepreneurs (gray area). On the other hand, the graph on the right shows the equilibrium when the project is financed with short term debt. Here the surplus of entrepreneurs is clearly reduced but, there is positive surplus for the lenders. Thus, lenders prefer this equilibrium.

What can we say from a social point of view? It should be straightforward to see that equilibrium $E_0$, i.e. the one with long term debt is socially optimal. On one hand, in the first scenario, the project takes place and it generates new resources with probability 1, while in the second it only generates resources with probability $\pi$: for the latter case, a planner intervention could improve both agents by reallocation of resources. Moreover, notice that in equilibrium $E_0$ the invested capital equals the optimal level of the entrepreneurs. On the other hand, when the project is financed with short term debt, $K$ is smaller than the optimal level of capital, even though lenders have available resources: in this case, there in under-investment in the economy.
Figure 2 also shows this point. The graph on the left shows the total surplus of the project when financed by long term debt (the gray area). On the other hand, the graph on the right shows the surplus when the project is financed by short term debt. There is a significant dead-weight loss (dark gray area) due to the possibility of crises and the corresponding reduction in investment. Clearly, from the surplus areas we can see long term debt is socially preferred.

### 4 Departing from the Competitive Bond Market

The model presented in the previous sections has two equilibria. One is preferred by the lenders (E1) and the other preferred by the entrepreneurs (E0). Unless we depart from the competitive bond market, we cannot say much more. In particular, we cannot say which of the two equilibria actually occur.

**Coordinated Entrepreneurs**

Let us first assume that entrepreneurs can coordinate to assure their preferred outcome. How do entrepreneurs coordinate? The natural way would be through the government. The government can impose taxes on short term debt, or can act as a lender of last resort.

Coordinated entrepreneurs would maximize their utility subject to Equation 4 to Equation 6. Clearly, the choice of entrepreneurs is to be in S, i.e. to run safe projects.

Note that despite coordination, the outcome is the same as in the competitive bond market equilibrium. This is due to the perfectly elastic supply of funds under long term financing.

**Coordinated Lenders**

The first way of coordination that one would think of is perfect coordination. Lenders will aim to extract monopoly profits from entrepreneurs. The outcome in this scenario would be long term borrowing (no risk) and they would charge an interest rate higher than the world’s risk free. Moreover, the capital inflows would be very low. Formally, lenders maximize their problem (See Section 2.3)
subject to entrepreneurs’ demand $f'(K) = 1 + r$.

It is easy to see that this is the classic cartel problem of coordination. Unless there is some technology to avoid defection, this outcome will not be sustained in equilibrium: lenders have incentives to deviate. Since each lender is negligible and does not affect the market interest rate, she will decide to invest more than the monopoly quantity.

Without a technology to deter defection, the coordination itself should assure that lenders will not deviate. This implies that the contract has to consider not only entrepreneurs demand but also that lenders cannot do better lending directly to entrepreneurs. Therefore, the contract should yield a Nash Equilibrium.

The only Nash equilibria in this model are the ones obtained in the previous sections: one with short term debt ($E_1$) and the other with long term debt ($E_0$). Since coordinated lenders that can not avoid defection should choose a Nash equilibrium, they choose the one in which they are better off: $E_1$.

We think that this scenario resembles the late 90’s financial markets situation (high levels of short term debt). However, it is difficult to think of a coordination as described above. On the other hand, we did observed that a significant fraction of the Asian flows came through financial intermediaries, such as hedge funds. We will present a model of optimal debt maturity with a hedge fund and show that it replicates the imperfect coordination result.

4.1 Equilibrium with Hedge Funds

Nowadays, most international investments are made through some type of financial intermediary, e.g. investment banks, hedge funds, etc. In particular, several authors have stressed the importance of such agencies in triggering some recent crises such as the East Asian one.

Through a hedge fund, lenders will be able to guarantee the existence of the equilibrium preferred by them ($E_1$).

A hedge fund will be an agency that offers a particular contract to the lenders. A contract is composed of four items: a return rate $r$, an investment
amount $K$, the probability of repayment $q^{14}$, and a fee charged by the agency for managing the funds $\lambda$. In other words, define a contract as $H_j = \{r_j, K_j, q_j, \lambda_j\}$.

The timing is similar to the one in the competitive bond market. However, there is an extra period: $T = 0$ where hedge funds compete. Without loss of generality, we will assume there are two hedge funds competing for funds à la Bertrand.

- At $T = 0$, hedge funds compete to attract lenders’ resources fully specifying all characteristics of contracts.
- At $T = 1$, hedge funds and entrepreneurs negotiate the debt structure$^{15}$
- At $T = 2$, lenders decide to roll over or not the loans.
- At $T = 3$, production takes place and debts are repaid.

In order to attract lenders to invest through the hedge funds, these will necessarily offer contracts in which lenders have no incentives to deviate. In addition, they should meet entrepreneur’s demand of capital that will be given by $f'(K) = 1 + r$. Thus, hedge funds will only offer contracts that satisfy either $E_0$ or $E_1$.

Notice that to run or not is not a decision variable for the hedge fund, i.e. its existence does not avoid the possibility of a self fulfilling crisis$^{16}$. However, given that it is (fully) leveraged, and that the investors can always ask their money from the fund, whenever investors run on the hedge fund, the latter has to run on the entrepreneurs.

Formally, a particular hedge fund solves the following problem:

$$
\max \begin{cases} 
q\lambda_1[rK] & , \quad H_1 \succ_L H_2 \\
q\lambda_1[rK/2] & , \quad H_1 \sim_L H_2 \\
0 & , \quad H_1 \prec_L H_2 
\end{cases}
$$

}s.t.

---

$^{14}$Choosing $q$ is equivalent to choose between long term debt ($q = 1$) or short term debt ($q = 1 - \pi$).

$^{15}$Notice that, since the contract was already fully specified in period 0 by the hedge fund and the lenders, the entrepreneur basically has no decision to take in this period. She can only decide to take or to reject the hedge fund’s offer.

$^{16}$In other words, we are assuming an open-end fund.
\[ V_{H_1} = \begin{cases} 
(1 - \pi) V(W + (1 - \lambda_1) rK) + \pi V(W - (1 - \varphi)K), & q = 1 - \pi \\
\frac{\pi}{1 - \pi} \frac{V'(W - (1 - \varphi)K)}{V(W) + (1 - \lambda_1) rK}, & q = 1 - \pi \\
0, & q = 1 
\end{cases} \]

\[ r = \begin{cases} 
\frac{\pi}{1 - \pi} \frac{V'(W - (1 - \varphi)K)}{V(W) + (1 - \lambda_1) rK}, & q = 1 - \pi \\
0, & q = 1 
\end{cases} \]

where \( H_1 \succ_L H_2 \) means lenders prefer (get higher utility) contract of hedge fund 1 than contract of hedge fund 2. Hedge funds are constrained by the entrepreneurs’ demand and the supply of funds by lenders\(^{17}\).

Clearly, the competition among hedge funds will drive the fee to zero. Moreover, it will imply the occurrence of Equilibrium \( E_1 \).

**Proposition 3** Assume there is free entry of hedge funds in the economy. Hedge funds do not charge any fee for managing the funds, i.e. \( \lambda = 0 \). Moreover, the unique equilibrium is: \( D = K < \bar{K} \) and \( r = \frac{\pi}{1 - \pi} \frac{V'(W - (1 - \varphi)K)}{V(W) + (1 - \lambda_1) rK} (1 - \varphi) \). \( (E1) \)

**Proof.**
See Appendix. ■

Obviously, the result of a zero transaction fee is not a realistic result. In reality, hedge funds do charge a fee for their operations. This happens basically because of imperfect information. Given that hedge fund has some information that is not accessible (or accessible at a high cost) to individual lenders, the latter are willing to pay some kind of fee. Another explanation would be that through the hedge fund, lenders can exploit increasing returns, sharing a possible high fee among many agents.

### 5 Concluding Remarks

This paper presents a model of optimal debt maturity. If long term projects are financed with short term debt there are possibilities of (self-fulfilling) liquidity crises. These crises cause very much damage to the global economy, specially to

\(^{17}\)Remember that lenders can have one of two supply schedules (See Equations 4 and 6) depending on the probability \( q \), which in turn depends on the debt maturity.
emerging market entrepreneurs, the borrowers. This is precisely the reason that makes so puzzling the existence of short term debt. If borrowers suffer from it, why don’t they avoid it?

The assumption of production complementarities is the novelty feature of the model. This assumption destroys the monopoly power that borrowers have on their risk of liquidity. If enough firms are financed with short term debt, the possibility of a liquidity crisis will affect the whole economy, including the firms with long term borrowing.

In this paper, we have shown a situation in which short term borrowing can be an equilibrium. Nevertheless, the assumptions of the paper do not exclude the possibility for long term borrowing as an equilibrium. Hence, we have multiple equilibria. The natural question to ask is What are the welfare effects of each equilibrium?

Long term borrowing ensures projects are completed and that the optimal amount of capital is invested. This is a Pareto efficient allocation. But, precisely for this reason, lenders will compete to invest their money, reducing the interest rates and, therefore, destroying any possibility to get part of the surplus the projects generate.

On the other hand, short term lending may produce the abandonment of profitable projects because of expectational shocks. This will produce lower investment and higher interest rates. Clearly this is not efficient and reduces borrowers welfare. But, surprisingly it improves lenders welfare. Short term borrowing has risk but has a higher return as well. Therefore, investing in short term debt enlarges the menu of assets for international investors.

The final part of the paper shows a possible rationale for the excessive short term debt observed in emerging markets. Given the positive effect on the foreign investors, they will try to coordinate in order to achieve the short term equilibrium. Thus, we model a different view of the financial markets, where there is no competitive bond market. Instead, there are financial intermediaries that compete with each other to manage lenders’ money. Once this stage is over, they negotiate with entrepreneurs. The result is that only short term debt can be an equilibrium.
References


